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HEAT EXCHANGE IN TURBULENT FLOW OF AN INCOMPRESSIBLE LIQUID IN A ROUGH TUBE WITH A CONSTANT WALL TEMPERATURE

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Turbulent flow of an incompressible liquid in a tube is considered as flow of an oriented liquid whose symmetry is determined by the director. The velocity profile and the temperature field in steady-state flow in a straight circular tube with a rough wall are determined within the framework of the model. The solutions found coincide with analogous solutions for a smooth tube for roughness equal to zero.

Introduction. Turbulent liquid flows in smooth and rough tubes noticeably differ in properties. Therefore, one usually considers them separately, thus emphasizing the significant character of differences. It is well known, e.g., that a logarithmic velocity profile near a smooth wall does not immediately follow from the universal velocity profile near a rough wall. However, it has turned out that one can consider both resistance and heat exchange in the case of flows in smooth and rough tubes within the framework of a single model [1, 2], properly formulating boundary conditions. Moreover, the corresponding solutions are also obtained for a smooth tube for zero roughness.

The present work seeks to prove this statement. The order of presentation is traditional: first the velocity profile is sought and later the problem on heat exchange is solved. The solutions obtained are compared to experimental results.

Velocity Profile. Within the framework of the model of [1, 2], we consider turbulent flow in a circular tube of not very large radius (permissible tube radii are refined below) as the motion of a specific medium whose local parameters are the averaged velocity u_i and the vector of local symmetry, i.e., the director n_i. In the case of an incompressible liquid the equations of motion of the medium in Cartesian coordinates x_i have the form

$$u_{i,i} = 0 \, , \tag{1}$$

$$\rho \dot{u}_i = p_{ij,j} + \rho f_i \, , \tag{2}$$

$$\rho J \ddot{n}_i = \beta_{ij,j} + g_i + \rho G_i \, . \tag{3}$$

Here the points above symbols denote the substantial time derivatives; summation from 1 to 3 over double subscripts is assumed.

The governing equations of the model are prescribed by the expressions [1, 2]

$$p_{ij} = -p\delta_{ij} + \sigma_{ij} + \tau_{ij} \, , \tag{4}$$

$$\sigma_{ij} = K n_{\alpha,i} \left(n_{j,\alpha} - n_{\alpha,j} + n_j n_\beta n_{\alpha,\beta} \right) , \tag{5}$$

$$\tau_{ij} = \mu_1 n_\alpha n_\beta e_{\alpha\beta} n_i n_j + \mu_4 e_{ij} \, , \tag{6}$$

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$$\beta_{ij} = \kappa_j n_i + K\,(n_{i,j} - n_{j,i} - n_j n_\alpha n_{i,\alpha})\,, \tag{7}$$

$$g_i = \chi n_i - (\kappa_\beta n_i)_{,\beta} + K n_\alpha n_{\beta,\alpha} n_{\beta,i}\,, \tag{8}$$

$$n_{i,j} = \frac{\partial n_i}{\partial x_j}\,, \quad e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

Since the properties of the liquid near a solid wall are determined by the vortex structure of the flow, the coefficients μ_1, μ_4, and K can be dependent on parameters globally characterizing flow, e.g., on the Reynolds number [2, 3]. The equations presented suffice to determine the velocity profile at a constant liquid temperature. The heat-transfer equation will be written below.

Let a viscous incompressible liquid (fluid) move in an infinite straight circular tube of radius R in the regime of steady-state turbulent flow. The problem on determination of the velocity profile will be solved in cylindrical coordinates r, φ, x with the x axis along the tube axis in the direction of flow. We assume that the coefficients μ_1, μ_4, and K in the flow in question are constant. Disregarding the external mass forces f_i and G_i in Eqs. (2) and (3), we will seek the velocity u_m and the director n_m in the form

$$u_x = u\,(r)\,, \quad u_r = u_\varphi = 0\,, \quad n_x = \cos\theta\,(r)\,, \quad n_r = \sin\theta\,(r)\,, \quad n_\varphi = 0\,. \tag{9}$$

Substitution of expressions (9) into Eqs. (1)–(8) with allowance for the assumptions made leads to equations necessary for solving the problem. The continuity equation (1) is satisfied identically. By virtue of arbitrariness of χ and κ_i in formulas (7) and (8), we set them equal to zero. As a result, from Eqs. (3) we obtain, for θ, the equation

$$\sin\theta\cos\theta\left(\theta'' + \frac{\theta'}{r}\right) - (2 - 3\cos^2\theta)\,\theta'^2 = 0\,. \tag{10}$$

Equations (2) and (4)–(6) in combination with (9) yield

$$\frac{1}{r}\frac{d\,(r\tau_{rr})}{dr} - \frac{\partial p}{\partial r} = 0\,, \quad \frac{1}{r}\frac{d\,(r\tau_{xr})}{dr} - \frac{\partial p}{\partial x} = 0\,; \tag{11}$$

$$\tau_{rr} = \mu_1 \sin^3\theta\cos\theta u'\,, \quad \tau_{xr} = \tau_{rx} = \left(\mu_1 \sin^2\theta\cos^2\theta + \frac{\mu_4}{2}\right)u'\,. \tag{12}$$

It follows from the first equation of (11), with account for the first formula of (12), that the partial derivative $\partial p/\partial r$ is independent of the coordinate x; therefore, the pressure distribution over the cross section of the tube has the form

$$p\,(x, r) = p_1\,(x) + p_2\,(r)\,, \tag{13}$$

and, as follows from the second equation of (11) and equality (13), we have $\partial p/\partial r$ = const. After the integration of the second equation of (11) and subsequent substitution of the second formula of (12) into it, we obtain an equation for the profile of longitudinal velocities u:

$$\left(\mu_1 \sin^2\theta\cos^2\theta + \frac{\mu_4}{2}\right)u' = -\frac{r}{R}\,\tau_{\rm w}\,. \tag{14}$$

As far as the structure itself is considered, the model in question does not include the laminar and transition layers of the generally accepted three-layer model [4, 5]; therefore, observing rigor, we should set boundary conditions at the upper boundary of the transition layer. However, the comparison made to experimental data [2, 3, 6] and the comparison that will be made below show that in the case of a smooth wall we can set boundary conditions directly on the tube wall. Boundary conditions for rough tubes will be formulated analogously.

Let k_e be the equivalent roughness of the tube. We take that the roughness is reckoned from the cylindrical surface $r = R$, i.e., the bases of the prominences of the equivalent granular roughness lie at distance R from the tube axis. Just as in the semiempirical theory of flows in rough tubes [4, 5], we will assume that the longitudinal velocity vanishes for $r = R - (k_e/30)$. Furthermore, the angle θ is assumed to be equal to zero on this surface. Thus, boundary conditions have the form

$$\theta\big|_{r=R-(k_e/30)} = 0\ ,\quad u\big|_{u=R-(k_e/30)} = 0\ . \tag{15}$$

When $k_e = 0$ conditions (15) become boundary conditions on a smooth wall [6].

The first integration of Eq. (10) yields

$$r \sin\theta \cos^2\theta\theta' = -\,bR\ . \tag{16}$$

The value of the constant of integration b can be determined by the condition at the boundary $r = r_0$ to which the wall vortex structure extends [2, 6]:

$$b = -\,(r_0/R) \sin\theta_0 \cos^2\theta_0\theta_0'\ . \tag{17}$$

The angle θ grows with distance from the wall; therefore, in these coordinates, we have $\theta' < 0$ and consequently $b > 0$.

Integrating Eq. (16) with the first boundary condition of (15), we obtain

$$\cos^3\theta = 1 + 3bR \ln\frac{\xi}{1-\zeta_e}\ ,\quad \xi = \frac{r}{R}\ ,\quad \zeta_e = \frac{k_e}{30R}\ . \tag{18}$$

The quantity ζ_e is small compared to unity and we can disregard it in formula (18). Near the tube wall, we replace the function $\ln\xi$ by the first term of its expansion in the Taylor series, i.e., by the function $(\xi - 1)$. As a result, we have

$$\cos\theta = [1 - 3bR\,(1-\xi)]^{1/3}\ . \tag{19}$$

Integration of Eq. (14) with account for formula (19) and the second boundary condition of (15) yields the velocity profile sought in the form

$$u = Au_*\,[\Phi(\xi) - \Phi(\xi_e)]\ ,\quad \Phi(\xi) = F(t(\xi))\ ,\quad \xi_e = 1 - \zeta_e\ , \tag{20}$$

$$F(t) = \frac{3bR-1}{2\gamma^2-1}\left(\sqrt{\gamma^2-1}\,\arctan\frac{t}{\sqrt{\gamma^2-1}} + \frac{\gamma}{2}\ln\frac{\gamma-t}{\gamma+t}\right) +$$

$$+\,\frac{1+2\varepsilon}{4\,(2\gamma^2-1)}\ln\frac{\gamma^2-t^2}{t^2+\gamma^2-1} + \frac{1}{4}\ln\left|t^4 - t^2 - \varepsilon\right| + \frac{t^2}{2}\ ,$$

$$t = t(\xi) = [1 - 3bR\,(1-\xi)]^{1/3}\ ,\quad 2\gamma^2 = 1 + \sqrt{1+4\varepsilon}\ ,$$

$$u_* = \left(\frac{\tau_w}{\rho}\right)^{1/2}\ ,\quad A = \frac{\rho u_*}{3\mu_1 b^2 R}\ ,\quad \varepsilon = \frac{\mu_4}{2\mu_1}\ .$$

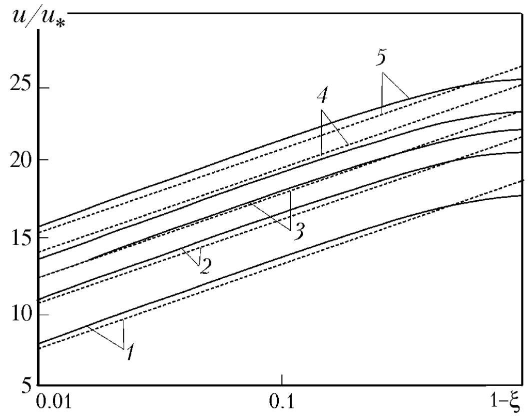


Fig. 1. Velocity profiles in rough tubes: 1) $k_e = R/60$; 2) $R/200$; 3) $R/400$; 4) $R/800$; 5) smooth tube.

If the distance from the wall $\zeta = 1 - \xi$ is short, we can substantially simplify the solution (20). For this purpose we will assume that the values of the model's parameters μ_1 and μ_4 are the same for flows in smooth and rough tubes. According to [2, 3, 6], the parameter γ is close to unity; therefore, we set $\gamma = 1$ in those terms of the solution where this is mathematically acceptable and leave unchanged the remaining terms; thereafter we expand all the terms in series in the parameter ζ, restricting ourselves to the first nonzero terms. As a result, we obtain the well-known "logarithmic law of wall" in rough tubes [4, 5]

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{1-\xi}{\zeta_e} = \frac{1}{\kappa} \left(\ln \frac{R-r}{k_e} + \ln 30 \right), \quad \kappa = \frac{2\mu_1 b}{\rho u_*} . \tag{21}$$

The velocity profile in a smooth tube is obtained from (20) for $k_e = 0$; near a solid wall, formula (21) yields, instead of it, the "logarithmic law" of a smooth wall [4, 5]

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{(R-r)\, u_*}{\nu} + C , \quad C = \frac{1}{\kappa} \ln \frac{\nu b}{(\gamma - 1)\, u_*} . \tag{22}$$

Thus, the solution (20) is the velocity profile for both rough and smooth tubes. Figure 1 plots the velocity profiles calculated from formula (20) (solid curves) and from the empirical formulas (21) and (22) (dashed curves) in air flow in a tube of diameter d = 100 mm and Re = 200,000. The calculated values of the parameters in formula (20) are $\rho = 1.205$ kg/m^3, $\nu = 1.5 \cdot 10^{-5}$ m^2/sec, $b = 4.83$ m^{-1}, $\mu_1 = 0.047 u_*$ Pa·sec, and $\mu_4 = 1.85 \cdot 10^{-6}$ Pa·sec. The values of the parameters b, μ_1, and μ_4 are the same as those obtained in [2, 3, 6] by comparing to experimental data in pressure and nonpressure air motions near a smooth wall. In the empirical universal formulas (21) and (22), we have $\kappa = 0.4$ and $C = 5.5$.

The calculated curves (20) and the empirical curves (21) and (22) are, apparently, close everywhere except for the region adjacent to the tube axis. As the roughness decreases, profiles 1–4 approach profile 5 in a smooth tube.

Heat Exchange. We find the steady-state temperature distribution in a semiinfinite tube $x \geq 0$ with a constant wall temperature in steady-state flow of a liquid with the velocity profile (20). In the adopted coordinate system r, φ, x, the temperature T is sought in the form

$$T = T(r, x) . \tag{23}$$

In determining the heat-flux density q_m, we take into account that the moving medium has a local symmetry prescribed by the director n_m; therefore, with account for formula (23), the Fourier law is written as [7]

$$q_r = -(\lambda_0 + \lambda_1 n_r^2)\frac{\partial T}{\partial r} - \lambda_1 n_r n_x \frac{\partial T}{\partial x},$$
$$q_\varphi = -\lambda_1 n_\varphi\left(n_r\frac{\partial T}{\partial r} + n_x\frac{\partial T}{\partial x}\right), \quad q_x = -\lambda_1 n_x n_r \frac{\partial T}{\partial r} - (\lambda_0 + \lambda_1 n_x^2)\frac{\partial T}{\partial x}. \tag{24}$$

When the flow regime is fixed, we will assume the coefficients λ_0 and λ_1 to be constant.

We replace the projections of n_m by their expressions (9) by the angle θ. The equation of propagation of heat is obtained by substitution of expressions (24) thus changed for q_m into the general equation of propagation of heat for a moving medium in cylindrical coordinates [8]

$$(\lambda_0 + \lambda_1 \sin^2\theta)\frac{\partial^2 T}{\partial r^2} + \left(\frac{\lambda_0 + \lambda_1 \sin^2\theta}{r} + \lambda_1 \sin 2\theta\theta'\right)\frac{\partial T}{\partial r} = \rho c_p u(r)\frac{\partial T}{\partial x}. \tag{25}$$

In solving the problem, we will use the dimensionless variables

$$\Theta = \frac{T - T_{\mathrm{w}}}{T_0 - T_{\mathrm{w}}}, \quad \xi = \frac{r}{R}, \quad X = \frac{x}{R}. \tag{26}$$

Substitution of the velocity profile (20) and the variables (26) into Eq. (25) reduces it to the calculated form

$$\frac{\partial^2\Theta}{\partial\xi^2} + \Psi_1(\xi)\frac{\partial\Theta}{\partial\xi} = \Psi_2(\xi)\frac{\partial\Theta}{\partial X},$$
$$\Psi_1(\xi) = \frac{1}{\xi} - \frac{2\lambda_1 bR}{t(\xi)\,[\lambda_0 + \lambda_1(1 - t^2(\xi))]}, \quad \Psi_2(\xi) = \frac{\rho c_p u_* AR\,(\Phi(\xi) - \Phi(\xi_{\mathrm{e}}))}{\lambda_0 + \lambda_1(1 - t^2(\xi))}, \tag{27}$$

where $t(\xi)$ and $\Phi(\xi)$ are the functions determined in (20). Strictly speaking, the use of the profile (20) in Eq. (25) over the entire cross section of the tube is not quite justified, since formula (20) holds only for the wall region. However, judging from the plots in Fig. 1, we are not making a big mistake by acting in this manner.

In formulating the boundary conditions, we disregard the change in the temperature in the layer of thickness $k_{\mathrm{e}}/30$ and will assume that $T = T_{\mathrm{w}}$ on the cylinder surface $r = \xi_{\mathrm{e}}R$. Suppose we have $T_0 = \mathrm{const}$ at the inlet $x = 0$ of the computational domain. Then the boundary conditions for Eq. (27) will be written in the form

$$\Theta(\xi, 0) = 1, \quad \Theta(\xi_{\mathrm{e}}, X) = 0. \tag{28}$$

To solve Eq. (27) we use the approximate Galerkin method [9], seeking the function $\Theta(\xi, X)$ in the form

$$\Theta(\xi, X) = \sum_{k=1}^{n} g_k(X)\,\varphi_k(\xi), \tag{29}$$

and taking, as the basis functions $\varphi_k(\xi)$ satisfying the second boundary condition (28), the functions

$$\varphi_k(\xi) = [\Phi_0(\xi) - \Phi_0(\xi_{\mathrm{e}})]\cos((k-1)\pi\xi), \quad k = 1, 2, ..., n,$$
$$\Phi_0(\xi) = 2(3bR - 1)\ln\frac{\gamma - t(\xi)}{\gamma + t(\xi)} + \ln\frac{\gamma^2 - t^2(\xi)}{t^2(\xi) + \gamma^2 - 1} + \ln\left|t^4(\xi) - t^2(\xi) - \varepsilon\right| + 2t^2(\xi). \tag{30}$$

The functions $g_k(X)$ are sought from the system of differential equations

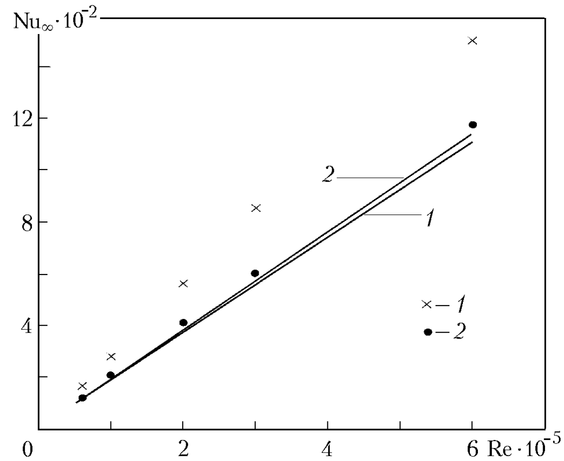


Fig. 2. Limiting Nusselt number vs. Reynolds number in rough tubes for air: 1) $k_e = R/60$; 2) $R/252$; lines, calculations from formulas (29)–(34); points, Nunner's experimental results [11].

$$\frac{dg_i(X)}{dX} = \sum_{j=1}^{n} b_{ij} g_j(X) , \quad i = 1, 2, ..., n , \tag{31}$$

whose coefficients b_{ij} are the elements of the matrix B determined by the formulas

$$B = M^{-1} L , \quad L = (l_{ij})_1^n , \quad M = (m_{ij})_1^n , \tag{32}$$

$$l_{ij} = \int_0^{\xi_e} \varphi_i(\xi) \left[\varphi_j''(\xi) + \Psi_1(\xi) \varphi_j'(\xi) \right] \xi d\xi , \quad m_{ij} = \int_0^{\xi_e} \varphi_i(\xi) \varphi_j(\xi) \Psi_2(\xi) \xi d\xi .$$

Let β_i be the eigenvalues of the matrix B, $H = (h_{ij})_1^n$ be the matrix of normalized eigenvectors (columns) of the matrix B, and $S = (s_{ij})_1^n$ be the matrix with elements (the superscript n is equal to the number of terms in the sum (29))

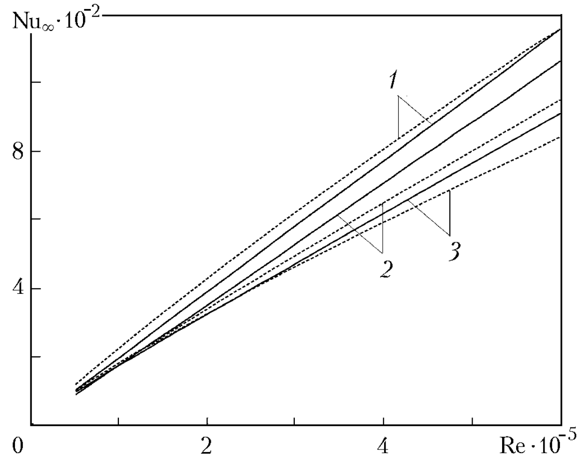
$$s_{ij} = \int_0^{\xi_e} \varphi_i(\xi) \varphi_j(\xi) \xi d\xi .$$

Then the solution of system (31) with the first boundary condition of (28) satisfied according to the Galerkin method is represented in the form

$$g_k(X) = \sum_{i=1}^{n} M_i h_{ki} \exp(\beta_i X) , \quad M_i = \sum_{j=1}^{n} d_{ij} l_j , \quad l_i = \int_0^{\xi_e} \varphi_i(\xi) \xi d\xi , \tag{33}$$

where d_{ij} is the element of the matrix $D = (SH)^{-1}$.

Formulas (29)–(30) with a prescribed n enable us to solve approximately the problem on temperature distribution in the flow. Based on the determination of the local Nusselt number Nu [8, 10], we can subsequently compute it from the formula



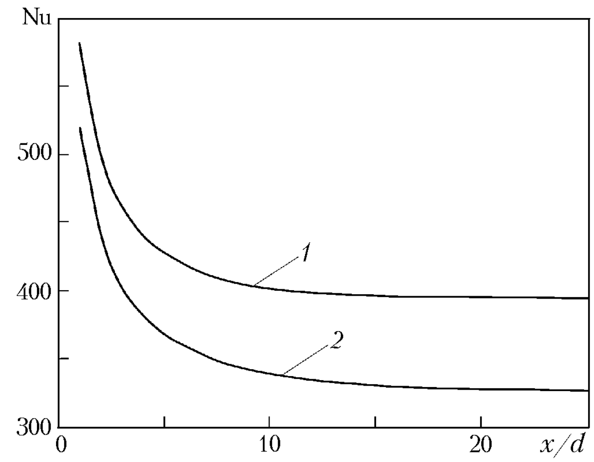


Fig. 3. Limiting Nusselt number vs. Reynolds number in rough tubes for air: 1) $k_e = R/200$; 2) $R/800$; 3) smooth tube; solid curves, calculation from formulas (29)–(34); dashed curves: 1, 2, plots of the function (36), 3) of the function (35).

Fig. 4. Change in the local Nusselt number on the initial thermal portion of rough (1) and smooth (2) tubes as a function of x/d (d = 100 mm, Re = 200,000, and $k_e = R/200$).

$$\mathrm{Nu} = -\frac{2\lambda_0}{\lambda\overline{\Theta}}\left(\frac{\partial\Theta}{\partial\xi}\right)_{\xi_e}, \quad \overline{\Theta}(X) = \frac{2}{w}\int_0^{\xi_e}\Theta(\xi, X)\,u(\xi)\,\xi d\xi\,. \tag{34}$$

To compare to experimental results we realized the solution under specific conditions (liquid–air: ρ = 1.205 kg/m^3, $\nu = 1.5 \cdot 10^{-5}$ m^2/sec, $\lambda = 2.57 \cdot 10^{-2}$ W/(m·K), c_p = 1002 J/(kg·K), and Pr = 0.72). The coefficients of the model were the same as those in experiments with smooth tubes [3, 6]: $\mu_1 = 0.047u_*$ Pa·sec, $\mu_4 = 1.85 \cdot 10^{-6}$ Pa·sec, $\lambda_0 = 0.28Ru_*$ W/(m·K), and $\lambda_1 = 46.5u_*$ W/(m·K). The calculations were carried out for n = 25.

As is seen from formula (19), for a fixed value of b in tubes whose diameter is larger than a certain b-dependent value, the velocity profile (20) can fill the tube cross section only partially. Taking b = 4.83 m^{-1}, we obtain that the profile (20) fills the cross section in tubes of radius $R \le 69$ mm. The calculations show that for a fixed liquid, the Nusselt number Nu at large X values is dependent just on the Reynolds number Re and the relative roughness k_e/R; therefore, the results given below hold for all tubes whose diameters lie in the above range.

Figure 2 give results of calculation of the limiting local Nusselt number Nu_∞ from formulas (29)–(40) for X = 200 and Nunner's experimental results from [11]. Figure 3 compares the calculated Nu_∞–Re curves to the plots of empirical dependences for a smooth tube [10]

$$\mathrm{Nu}_\infty = 7.6 - \frac{3.6}{\log \mathrm{Re}} + 0.0096\mathrm{Re}^{0.87}\mathrm{Pr}^{0.605} \tag{35}$$

and for a rough tube [12]

$$\mathrm{Nu}_\infty = \mathrm{Pr}\,\mathrm{Re}\,\frac{f}{8}\left[1 + \left(5.19k_*^{0.2}\mathrm{Pr}^{0.44} - 8.48\right)\sqrt{\frac{f}{8}}\right]^{-1}, \tag{36}$$

$$f = \left(0.88 \ln \frac{R}{k_e} + 1.74\right)^{-2}, \quad k_* = \frac{k_e u_*}{\nu}.$$

The calculated Nu – (x/d) curves on the initial thermal portion of rough and smooth tubes are presented in Fig. 4.

Conclusions. An analysis of the calculated results compared to experimental data enables us to draw a number of conclusions. In tubes with moderate and small roughnesses ($k_e/R/200$) up to Re = 600,000 the calculated Nu–Re curves are in quite satisfactory qualitative and quantitative agreement with the experimental data. An increase in the coefficient of heat transfer in rough tubes compared to smooth tubes for all Reynolds numbers is evident from Figs. 3 and 4. As the roughness decreases, the Nu–Re curves approach analogous dependences for smooth tubes. Calculations for rough and smooth tubes have been carried out from the same equations and the same formulas and values of the constants determining b, μ_1, μ_4, λ_0, and λ_1 and differing only in boundary conditions for Eqs. (14) and (27). Therefore, we can state that the processes of heat exchange in tubes with a small roughness and in smooth tubes are close in nature and are quite adequately described by this model. When the roughness is large (see Fig. 2), the disagreement between the calculated and experimental data is significant. In such tubes, the mechanism of heat transfer near the wall, apparently, becomes different and is not covered by this model.

NOTATION

A, coefficient of (20); a, thermal diffusivity, m^2/sec; B, matrix of (32) with elements b_{ij}; b, constant of integration (16), 1/m; C, constant of the law of wall (22); c_p, specific heat at constant pressure, J/(kg·K); D, matrix with elements d_{ij}; d, tube diameter, m; e_{ij}, strain rates, 1/sec; f, coefficient of resistance of a tube; f_i, density of the mass force, m/sec^2; G_i, density of the external generalized mass force, m^2/sec^2; g_i, density of the internal generalized body force, Pa; $g_k(X)$, functions (29); h_{ij}, elements of the matrix H; H, matrix of normalized eigenvectors of the matrix B; J, density of the moment of inertia in rotation of the director, m^2; K, coefficient of the model, kg·m/sec^2; k_e, equivalent roughness, mm; k_*, constant of (36); L and M, matrices of (32); l_i and M_i, quantities of (33); Nu = $\alpha d/\lambda$, Nusselt number; Nu$_\infty$, limiting Nusselt number; n, number of the basis functions of (29); n_i, director; Pr = ν/a, Prandtl number; p, pressure, Pa; p_{ij}, stresses, Pa; q_m, heat-flux density, W/m^2; R, tube radius, m; Re = wd/ν, Reynolds number; r, distance from the tube axis, m; r_0, distance from the tube axis to the boundary of the wall vortex layer, m; r, φ, x, cylindrical coordinates; T, local temperature, K; T_0, temperature at the boundary x = 0, K; T_w, temperature on the tube wall, K; t, parameter in (20) and in what follows; u, longitudinal velocity in the tube, m/sec; u_i, local velocity in the flow, m/sec; u_*, dynamic velocity, m/sec; w, average velocity, m/sec; X, dimensionless longitudinal coordinate; x_i, Cartesian coordinates; α, heat-transfer coefficient, W/(m^2·K); β_i, eigenvalues of the matrix B; β_{ij}, generalized stresses, Pa·m; γ, constant of (20); δ_{ij}, Kronecker symbol; $\varepsilon = \mu_4/(2\mu_1)$, ratio of the model's coefficients; $\zeta = 1 - \xi$, dimensionless distance from the wall; ζ_e, dimensionless roughness of (18); Θ, dimensionless temperature of (26); $\overline{\Theta}$, dimensionless mass-mean temperature; θ, angle between the director and the x axis; θ_0, value of the angle θ for $r = r_0$; κ, von Kármán constant; κ_i, arbitrary vector (7) and (8), Pa·m; λ, molecular thermal conductivity, W/(m·K); λ_0 and λ_1, anisotropic turbulent thermal conductivities, W/(m·K); μ_1 and μ_4, coefficients of anisotropic turbulent viscosity, Pa·sec; ν, kinematic viscosity, m^2/sec; ξ, dimensionless coordinate of (18); ρ, density, kg/m^3; σ_{ij} and τ_{ij}, stresses of (4)–(6), Pa; τ_w, modulus of tangential stress on the wall, Pa; τ_{rr} and τ_{xr}, turbulent viscous stresses, Pa; φ, coordinate in the cylindrical coordinate system; $\Phi(\xi)$, function of (20); $\Phi_0(\xi)$, function of (30); χ, arbitrary scalar quantity of (8). Subscripts and superscripts: i, j, k, n, α and β, natural numbers; m, coordinates r, φ, x; ($'$) and ($''$), derivatives with respect to r; w, wall; e, equivalent.

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